

Department of Mathematics
Year 12 Scheme of Work – Mechanics



$$3 - 2 = 1 \quad \rightarrow \quad \sin^2\theta + \cos^2\theta = 1 \quad \rightarrow \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2} dt = 1$$

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Chapter/Objectives	Teaching Points
<u>STATISTICAL SAMPLING</u>	
<p><u>Chapter 1 – Statistical Sampling (Part A Sampling Terminology)</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • understand and be able to use the terms ‘population’ and ‘sample’; • know how to use samples to make informal inferences about the population; • be able to describe advantages and disadvantages of sampling compared to census. 	<p>This section is a great opportunity to introduce the large data set to look at a population of data and discuss reasons for sampling from it.</p> <p>Students will be expected to be able to comment on the advantages and disadvantages associated with a census and a sample.</p> <p>Discuss in context the meanings of populations and samples. Look at data from populations and samples, initially using data from the sample to make inferences about the population before then checking the data for the population.</p> <p>Discuss the advantages and disadvantages of sampling making sure to include time, cost etc.</p> <p>Ensure students are given the opportunity, and are able, to give full and thorough answers within the context of the question.</p>
<p><u>Chapter 1 – Statistical Sampling (Part B Sampling Techniques)</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • understand and be able to use sampling techniques; • be able to describe advantages and disadvantages of sampling techniques; • be able to select or critique sampling techniques in the context of solving a statistical problem; • understand that different samples can lead to different conclusions about the 	<p>Students will also be expected to be familiar with different types of sampling including simple random, stratified, systematic, quota and opportunity (convenience) sampling.</p> <p>Students will gain a more thorough understanding of the types of sampling if the advantages and disadvantages alongside the method used for each type are understood. They will then be more able to select an appropriate technique for a given statistical problem and be able to critique a technique which has been used.</p> <p>Give students the opportunity to use the techniques they learn about on the large data set.</p>

<u>MEASURES OF LOCATION AND SPREAD</u>	
<p><u>Chapter 2 – Measures of Location, Spread and Coding</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • be able to calculate measures of location, mean, median and mode; • be able to calculate measures of variation, standard deviation, variance, range and interpercentile range; • be able to interpret and draw inferences from summary statistics. 	<p>The calculation of the mean, median and mode should be recapped from GCSE however the focus now is on students using calculators to do the calculations. Check understanding of the terminology and teach calculator methods.</p> <p>Students require an understanding of measures of variation too and should be able to use their calculators to calculate the variance and standard deviation. They should be able to use the statistic $S_{xx} = \sum(x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$. Students are expected to use standard deviation = $\sqrt{\frac{S_{xx}}{n}}$ but equivalents including spreadsheet formula ($s = \sqrt{\frac{S_{xx}}{n}}$) will be accepted.</p> <p>The data may be discrete or continuous, grouped or ungrouped, and students need to be able to interpret these summary statistics clearly and be able to make inferences from them. Significance tests will not be expected.</p> <p>Coding for both mean and standard deviation needs to be covered. Be clear that students need to be able to uncode both mean and standard deviation. Emphasise that the standard deviation is unaffected by the addition or subtraction of constants.</p> <p>Students are expected to be able to use linear interpolation to calculate percentiles from grouped data.</p>

Chapter/Objectives	Teaching Points
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<u>REPRESENTATIONS OF DATA</u>	
<p><u>Chapter 3 – Representation of Data (Outliers, Box Plots, Cumulative Frequency and Histograms)</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • know how to interpret diagrams for single variable data; • recognise and interpret possible outliers in data sets and statistical diagrams; • be able to select or critique data presentation techniques in the context of a statistical problem; • be able to clean data, including dealing with missing data, errors and outliers. 	<p>Students should be familiar with and be able to interpret histograms, frequency polygons, box and whisker plots and cumulative frequency diagrams. These should have been covered at GCSE but it is worth a recap for consistency of methods. Also cover calculating summary statistics from diagrams, including the mean and standard deviation from a histogram.</p> <p>Outliers will need to be identified and interpreted from data sets and statistical diagrams. Any rules to be used will be given in the question, for example $Q_1 - 1.5 \times IQR$, $Q_3 + 1.5 \times IQR$.</p> <p>Students will be expected to select an appropriate diagram or critique the choice of one which is used. They should also be able to clean data by identifying possible outliers (box plots and scatter diagrams). They may also be asked to fill in missing data using a regression line.</p>

<u>CORRELATION AND LINEAR REGRESSION</u>	
<p><u>Chapter 4 – Correlation & Linear Regression</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • know how to interpret scatter diagrams and regression lines for bivariate data; • recognise the explanatory and response variables; • be able to make predictions using the regression line and understand its limitations; • understand informal interpretation of correlation; • understand that correlation does not imply causation; • recognise and interpret possible outliers in data sets and statistical diagrams; 	<p>For bivariate data students should understand the terms explanatory and response variables and know where each is placed on the axes of a scatter diagram. This is particularly important as variables other than y and x could be used.</p> <p>Students are not expected to know, calculate or understand the regression line formula. Students will need to understand the use of interpolation when using a regression line equation to make predictions within the range of values of the explanatory variable and they need to understand the dangers of extrapolation (predictions outside the range), again variables other than y and x could be used.</p> <p>Students will be expected to describe the correlation on a scatter diagram in terms of positive, negative or no correlation and strong or weak but no calculations need to be made. Values from calculations will not be given for interpretation.</p>

Chapter/Objectives	Teaching Points
<u>PROBABILITY</u>	
<p><u>Chapter 5 – Probability</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • understand and be able to use mutually exclusive and independent events when calculating probabilities; • be able to make links to discrete and continuous distributions. 	<p>Tree and Venn diagrams should have been covered at GCSE but will need to be recapped as one way of looking at probabilities.</p> <p>The focus at this level is on independent and mutually exclusive events in probability calculations. Students should be confident in the definitions of both independent and mutually exclusive events and how to use their properties to solve real-life probability problems.</p> <p>Cover showing independence but be aware that the use of set notation is not required at AS level. At this level this is done by showing the product of the probabilities of two events gives the probability of both events occurring together. Understanding of conditional probability is not expected at AS level.</p> <p>Students do not need to be aware of probability density functions however they should understand that probability is represented by the area under a curve in a continuous distribution. This could be mentioned here and comparisons drawn by using the binomial model as a bar chart in the next unit.</p>

<u>STATISTICS DISTRIBUTIONS</u>	
<p><u>Chapter 6 – Using and Identifying Discrete Distributions</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> • understand and be able to use simple, discrete probability distributions, including the binomial distribution; • be able to identify the discrete uniform distribution; • be able to calculate probabilities using the binomial distribution. 	<p>Students will be expected to model real-world situations by using simple discrete probability distributions. They should know and be able to recognise a discrete uniform distribution; look at equally likely outcomes such as numbers on a dice.</p> <p>The only specific distribution students are expected to use as well as understand is the binomial distribution. Students will be expected to comment critically on how appropriate a given probability model may be for a situation.</p> <p>The notation $X \sim B(n, p)$ may be used, so you should ensure students are familiar with this from the outset. Make sure the properties of the binomial are clear for all students, so that they know a fixed number of trials is needed, there are only two possible outcomes per trial and the outcome of each trial is independent.</p> <p>Once the binomial distribution has been introduced link back to thinking about probability being the area under a curve. Use a bar chart for discrete binomial distributions and show how this would smooth into a curve if it were a continuous distribution. Another teaching point for this concept of area could come from considering the discrete uniform distribution as bars of equal width; it looks like a rectangle, like the continuous uniform distribution.</p> <p>Students need to calculate probabilities using the binomial distribution for both individual and cumulative probabilities. Calculator use is expected for all of this, so time needs to be spent making sure students are competent in the use of these calculator functions.</p> <p>The bar chart model mentioned earlier helps students distinguish between for example $P(X < 2)$ and $P(X \leq 2)$, also to understand $P(X \geq 6) = 1 - P(X \leq 5)$. Explain this is due to the binomial being a discrete distribution. This is essential when manipulating before using the calculator to find probabilities. Encourage students to shade the bars required to help with this understanding.</p> <p>Emphasise the importance of reading questions carefully. The probability of success can be worded negatively in the question for example ‘the probability of people failing their driving test first time is 0.6’.</p> <p>Students are not expected to be able to calculate the mean and variance of discrete random variables.</p>

Chapter/Objectives	Teaching Points
<u>STATISTICAL HYPOTHESIS TESTING</u>	
<p><u>Chapter 7 – Hypothesis Testing (Part A Language of Hypothesis Testing)</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> understand and be able to apply the language of statistical hypothesis testing, developed through a binomial model. 	<p>The concept of a hypothesis could be introduced initially by posing some hypotheses yourself. You may wish to make reference to the large data set again and say for example ‘the daily maximum temperature was higher in Hurn than Heathrow in May 1987’.</p> <p>Following this introduce the null and alternative hypotheses and their respective notation H_0 and H_1. Discuss how to move from statements like the one above to using the language of the binomial distribution in terms of looking at p, the probability of success.</p> <p>The focus of this sub-unit is the language used in terms of hypothesis testing, but a scenario must be set. You may wish to use an example like ‘the number of 6s thrown in 50 throws of a dice’, students could carry out this experiment and you could use their results to form a variety of tests which would cover all of the terminology without actually carrying out the tests. Make sure you save these examples to be tested in the next sub-unit.</p> <p>All of the terms from the keywords section should be thoroughly discussed and understood before attempting to carry out a hypothesis test.</p>
<p><u>Chapter 7 – Hypothesis Testing (Part B Hypothesis Tests with Binomial Distribution)</u></p> <p>By the end of the sub-unit, students should:</p> <ul style="list-style-type: none"> be able to conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context; understand that a sample is being used to make an inference about the population; appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis. 	<p>Once all the terminology that has been discussed in the previous sub-unit is fully understood, you can go back to the examples you used and conduct the hypothesis tests. Carry out the tests both by finding the critical value to compare with your test statistic and by finding the probability (p-value) of the test statistic and comparing it with the critical region. Ensure students are competent with both methods. Make sure hypotheses are always written clearly in terms of p, the probability of success.</p> <p>Spend time making sure that students can write clear and concise conclusions in the given context of the questions.</p> <p>When using a sample of data, ensure students understand, what it infers about the population itself.</p> <p>Type I errors are not part of the specification, but it is important that students understand what the significance level of a test actually means. Discuss carefully that rejecting the null hypothesis may actually be incorrect and the significance level is the probability of this. Also cover ‘the actual significance level of a test’ with students.</p>