Chapter/Objectives	Teaching Points

Department of Mathematics
Year 12 Scheme of Work – Pure

$3 - 2 = 1$ → $\sin^2\theta + \cos^2\theta = 1$ → $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2}\,dt = 1$

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ALGEBRAIC EXPRESSIONS

Chapter 1 - Algebraic Expressions (Algebra, Indices and Surds) By the end of the sub-unit, students should: • be able to perform essential algebraic manipulations, such as expanding brackets, collecting like terms, factorising etc; • understand and be able to use the laws of indices for all rational exponents; • be able to use and manipulate surds, including rationalising the denominator.	Recap the skills taught at GCSE Higher Tier (9-1). Emphasise that in many cases, only a fraction or surd can express the exact answer, so it is important to be able to calculate with surds. Ensure students understand that $\sqrt{a}+\sqrt{b}$ is *not* equal to $\sqrt{a+b}$ and that they know that $a^{\frac{m}{n}}$ is equivalent to $\sqrt[n]{a^m}$ and that a^{-m} is equivalent to $\frac{1}{a^m}$. Most students understand the skills needed to complete these calculations but make basic errors with arithmetic leading to incorrect solutions. Questions involving squares, for example $(2\sqrt{3})^2$, will need practice. Students should be exposed to lots of simplifying questions involving fractions as this is where most marks are lost in exams. Recap the difference of two squares $(x+y)(x-y)$ and link this to $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = x-y$, explaining the choice of term to rationalise the denominator**.** Provide students with plenty of practice and ensure that they check their answers.

QUADRATICS

Chapter 2 – Quadratics (Factorising, Solving, Graph & Discriminants) By the end of the sub-unit, students should: • be able to solve a quadratic equation by factorising; • be able to work with quadratic functions and their graphs; • know and be able to use the discriminant of a quadratic function, including the conditions for real and repeated roots; • be able to complete the square. e.g. $ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$; • be able to solve quadratic equations, including in a function of the unknown.	Lots of practice is needed as these algebraic skills are fundamental to all subsequent work. Students must become fluent, and continue to develop thinking skills such as choosing an appropriate method, and interpreting the language in a question. Emphasise correct setting out and notation. Students will need lots of practice with negative coefficients for x squared and be reminded to always use brackets if using a calculator. e.g. $(-2)^2$. Include manipulation of surds when using the formula for solving quadratic equations. [Link with previous sub-unit.] Where examples are in a real-life contexts, students should check that solutions are appropriate and be aware that a negative solution may not be appropriate in some situations. Students must be made aware that this sub-unit is about finding the links between completing the square and factorised forms of a quadratic and the effect this has on the graph. Use graph drawing packages to see the effect of changing the value of the '$+c$' and link this with the roots and hence the discriminant. Start by drawing $y = x^2$ and add different *x* terms followed by different constants in a systematic way. Then move on to expressions where the coefficient of x^2 is not 1.

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EQUATIONS AND INEQUALITIES	
Chapter 3 – Equations and Inequalities (Part A Simultaneous Linear & Quadratic Equations) By the end of the sub-unit, students should: • be able to solve linear simultaneous equations using elimination and substitution; • be able to use substitution to solve simultaneous equations where one equation is linear and the other quadratic.	Simultaneous equations are important both in future pure topics but also for applied maths. Students will need to be confident solving simultaneous equations including those with non-integer coefficients of either or both variables. The quadratic may involve powers of 2 in one unknown or in both unknowns, e.g. Solve $y = 2x + 3$, $y = x^2 - 4x + 8$ or $2x - 3y = 6$, $x^2 - y^2 + 3x = 50$. Emphasise that simultaneous equations lead to a pair or pairs of solutions, and that both variables need to be found. Make sure students practise examples of worded problems where the equations need to be set up. Students should be encouraged to check their answers using substitution. Sketches can be used to check the number of solutions and whether they will be positive or negative. This will be reviewed and expanded upon as part of the curve sketching topic. Use graphing packages or graphing Apps (e.g. Desmos or Autograph), so students can visualise their solutions e.g. straight lines crossing an ellipse or a circle.
Chapter 3 – Equations and Inequalities (Part B Linear and Quadratic Inequalities) By the end of the sub-unit, students should: • be able to solve linear and quadratic inequalities; • know how to express solutions through correct use of 'and' and 'or' or through set notation; • be able to interpret linear and quadratic inequalities graphically; • be able to represent linear and quadratic inequalities graphically.	Provide students with plenty of practice at expressing solutions in different forms using the correct notation. Students must be able to express solutions using 'and' and 'or' appropriately, or by using set notation. So, for example: $x < a$ **or** $x > b$ is equivalent to $\{x: x < a\} \cup \{x: x > b\}$ and $\{x: c < x\} \cap \{x: x < d\}$ is equivalent to $x > c$ **and** $x < d$. Inequalities may contain brackets and fractions, but these will be reducible to linear or quadratic inequalities. For example, $\frac{a}{x} < b$ becomes $ax < bx^2$. Students' attention should be drawn to the effect of multiplying or dividing by a negative value, this must also be taken into consideration when multiplying or dividing by an unknown constant. Sketches are the most commonly used method for identifying the correct regions for quadratic inequalities, though other methods may be used. Whatever their method, students should be encouraged to make clear how they obtained their answer. Students will need to be confident interpreting and sketching both linear and quadratic graphs in order to use them in the context of inequalities. Make sure that students are also able to interpret combined inequalities. For example, solving
	$ax + b > cx + d$ $px^2 + qx + r \geq 0$ $px^2 + qx + r < ax + b$ and interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$. When representing inequalities graphically, shading and correctly using the conventions of dotted and solid lines is required. Students using graphical calculators or computer graphing software will need to ensure they understand any differences between the conventions required and those used by their graphical calculator.

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GRAPHS AND TRANSFORMATIONS	
Chapter 4 – Graphs and Transformations (Part A Cubic, Quadratic & Reciprocal) By the end of the sub-unit, students should: • understand and use graphs of functions; • be able to sketch curves defined by simple equations including polynomials; • be able to use intersection points of graphs to solve equations.	Students should be familiar with the general shape of cubic curves from GCSE (9-1) Mathematics, so a good starting point is asking students to identify key features and draw sketches of the general shape of a positive or negative cubic. Equations can then be given from which to sketch curves. Quartic equations will be new to students and they may benefit from initially either plotting graphs by hand or using a graphical calculator or graphing software to look at the shape of the curve. Cubic and quartic equations given at this point should either already be factorised or be easily simplified (e.g. $y = x^3 + 4x^2 + 3x$) as students will not yet have encountered algebraic division. The coordinates of all intersections with the axes will need to be found. Where equations are already factorised, students will need to find where they intercept the axes. Repeated roots will need to be explicitly covered as this can cause confusion. Students should also be able to find an equation when given a sketch on which all intersections with the axes are given. To do this they will need to be confident multiplying out multiple brackets. Reciprocal graphs in the form $y = \frac{a}{x}$ are covered at GCSE but those in the form $y = \frac{a}{x^2}$ will be new. When sketching reciprocal graphs such as $y = \frac{a}{x}$ and $= \frac{a}{x^2}$, the asymptotes will be parallel to the axes. Intersecting points of graphs can be used to solve equations, a curve and a line and two curves should be covered. When finding points of intersection students should be encouraged to check that their answers are sensible in relation to the sketch.
Chapter 4 - Graphs and Transformations (Part B Transforming Graphs) By the end of the sub-unit, students should: • understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$; • be able to sketch the result of a simple transformation given the graph of any function $y = \mathrm{f}(x)$.	Transformations to be covered are: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ and $y = \mathrm{f}(ax)$. Students should be able to apply one of these transformations to any of the functions listed and sketch the resulting graph: quadratics, cubics, quartics, reciprocals, $y = \frac{a}{x^2}$, $\sin x$, $\cos x$, $\tan x$, e^x and a^x. Students will need to be able to transform points and asymptotes both when sketching a curve and to give either the new point or the equation of the line. Given a curve or an equation that has been transformed students should be able to state the transformation that has been used. Links can be made with sketching specific curves. Students should be able to sketch curves like $y = (x - 3)^2 + 2$ and $y = \frac{2}{x-3} + 2$

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STRAIGHT LINE GRAPHS

Chapter 5 - Straight Line Graphs (Parallel, Perpendicular, Length and Area Problems) By the end of the sub-unit, students should: • understand and use the equation of a straight line; • know and be able to apply the gradient conditions for two straight lines to be parallel or perpendicular; • be able to find lengths and areas using equations of straight lines; • be able to use straight-line graphs in modelling.	Students should be encouraged to draw sketches when answering questions or, if a diagram is given, annotate the diagram. Equations can be given or asked for in the forms $y = mx + c$ and $ax + by + c = 0$ where a, b and c are integers. Students will need to be familiar with both forms, so questions should be asked where different forms are given or required in the answer. Given either form, students should be able to find the intercepts with the axes and the gradient. The x-intercept often causes students more difficulty, so will need more practice, but is useful for sketches and questions involving area or perimeter. Students should be able to find the equation of a line given the gradient and a point, either the formula $y - y_1 = m(x - x_1)$ can be used or the values substituted into $y = mx + c$. To find the equation of a line from two points the gradient can be found and then one of the previous two methods used or the formula $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ can be used. If this formula is used, care needs to be taken to ensure that the y-values are substituted into the correct places and that negative signs are taken into account. It should be emphasised that in the majority of cases, the form $y - y_1 = m(x - x_1)$ is far more efficient and less prone to errors than other methods. The gradient conditions for parallel and perpendicular lines may be remembered from GCSE (9-1), but are still worth revising. They need to be well understood as they are used further when dealing with circles and in differentiation. Students should be able to identify whether lines are parallel, perpendicular or neither and find the equation of a parallel or perpendicular line when given a point on the line. The length of a line segment is found by using Pythagoras' theorem, which can be written as the formula $d = \sqrt{(x - x_1)^2 + (y_2 - y_1)^2}$. This can be linked to proof with students being encouraged to show how to go from Pythagoras to the formula. Answers to length and distance questions are likely to be given in surd form, giving further practice in simplifying surds. Students should be encouraged to give answers in exact form unless specified otherwise. Make shapes using lines and the axes; students can then be asked to find the area or perimeter of composite shapes. Answers should be given in exact form to practise combining and simplifying surds. Real-life situations such as conversions can be modelled using straight-line graphs, this is likely to be familiar from GCSE (9-1) Mathematics. Students should also be familiar with finding the relationship between two variables and expressing this using the proportion symbol $\propto$ or using an equation involving a constant (k). This can be extended to straight-line graphs through the origin with a gradient of k. Students should be able to calculate and interpret the gradient.

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CIRCLES

Chapter 6 – Circles (Equation, Intersections, Tangents and Chords) By the end of the sub-unit, students should: • be able to find the midpoint of a line segment; • understand and use the equation of a circle; • be able to find points of intersection between a circle and a line; • know and be able to use the properties of chords and tangents.	Drawing sketches or annotating given diagrams will help students to understand the question in many cases and so should be encouraged. Students should be able to find the midpoint given two points from GCSE (9-1) Mathematics. This can be built upon to find the coordinate of a point given the midpoint and one of the end points. The midpoint can be used to find the perpendicular bisector, recapping the work from straight-line graphs. The equation of the circle $(x-a)^2+(y-b)^2=r^2$ can be derived from Pythagoras' theorem, giving students the opportunity to look at proof. Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa. Students should be familiar with the equations $x^2+y^2+2fx+2gy+c=0$ and $(x-a)^2+(y-b)^2=r^2$. 'Complete the square' method should be used to factorise the equation into the more useful form. Students will need practice within this context to ensure that they are confident with the algebraic manipulation needed, in particular mistakes are often made with the signs and forgetting the constant term. Circle theorems from GCSE (9-1) Mathematics can be used in questions so a quick recap could be useful and then they should be incorporated into questions. Examples of this include: finding the equation of the circumcircle of a triangle with given vertices; or finding the equation of a tangent using the perpendicular property of tangent and radius. Simultaneous equations can be used to find the points of intersection between a circle and a straight line. Students can also be asked to show that a line and circle do not intersect, for which the discriminant can be used. Finding intersections with the axes should also be covered.

ALGEBRAIC METHODS

Chapter 7 - Algebraic Methods (Division, Factor Theorem & Proof) By the end of the sub-unit, students should: • be able to use algebraic division; • know and be able to apply the factor theorem; • be able to fully factorise a cubic expression; • understand and be able to use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; • be able to use methods of proof, including proof by deduction, proof by exhaustion and disproof by counter-example.	When using algebraic division, only division by $(ax+b)$ or $(ax-b)$will be required. Different methods for algebraic division should be considered depending on students' prior experience and preferred ways of working. Whichever method is used, clear working out should be shown. Equations in which the coefficient of x or x^2 is 0 for example x^3+3x^2-4 or $2x^3+5x-20$ will need additional explanation and practice. Students should know that if $\text{f}(x)=0$ when $x=a$ then $(x-a)$ is a factor of $\text{f}(x)$. Questions in the form $(ax+b)$ should be covered. Where a negative is being substituted into the equation the distinction between $(-2)^2$ and -2^2 will be important especially when students are using a calculator as examiners often comment on the fact that students will sometimes evaluate $(-2)^2$ as -4. Factor theorem can be used to find an unknown constant. For example: Find a given that $(x-2)$ is a factor of x^3+ax^2-4x+6. Two conditions can also be given in order to form simultaneous equations to solve. When fully factorising a cubic, emphasis should be placed on choosing appropriate values. The final answer may need to be written as a factorised cubic or, alternatively, as the solutions to an equation which can then be used to sketch the curve. Students sometimes use the roots of a polynomial equation to help them factorise but this method must be used with care. Questions sometimes use the word 'hence' and so students must be careful which method they chose in these cases. This is an excellent opportunity to review curve sketching by asking students to give a sketch following factorisation. Students should be familiar with basic proofs from GCSE (9-1) Mathematics this knowledge can be built upon to look at the different types of proof. Students will need to understand how to set out each type of proof; the correct conventions in language and layout should be encouraged.

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BINOMIAL EXPANSION	
Chapter 8 - The Binomial Expansion By the end of the sub-unit, students should: • understand and be able to use the binomial expansion of $(a + bx)^n$ for positive integer n; • be able to find an unknown coefficient of a binomial expansion.	Students should initially be introduced to Pascal's triangle, which can be used to expand simple brackets. Students will need to be familiar with factorials and the $_n\mathrm{C}_r$ notation. Introduce the formal binomial expansion in the same way as the formula booklet and discuss the various terms to ensure all students understand. Setting out work clearly and logically will be invaluable in helping students to achieve the final answer and also to spot mistakes if necessary. Where there is a coefficient of x (other than 1) students will need to be reminded that the power applies to the whole term, not just the x, and that answers must be simplified appropriately. Negative and fractional coefficients will also need practice. The limitations of the binomial expansion should be discussed. Students should practice finding the coefficient of a single term, they should also be able to deal with setting up simple algebraic equations to find unknown constants. Use of the binomial expansion can be linked to basic probability and approximations. [Links can also be made with the statistics work in A level Mathematics.]

TRIGONOMTERIC RATIO AND GRAPH	
Chapter 9 - Trigonometric Ratios & Graphs By the end of the sub-unit, students should: • understand and be able to use the definitions of sine, cosine and tangent for all arguments; • understand and be able to use the sine and cosine rules; • understand and be able to use the area of a triangle in the form $\frac{1}{2}ab\ \sin\ C$; • understand and be able to use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.	Students should be shown the *x* and *y* coordinates of points on the unit circle can be used to give cosine and sine respectively. Use of trigonometric ratios will have been covered at GCSE (9-1) Mathematics; questions should now be focused more on multi-step problems and questions set in context. When using the sine rule the ambiguous case should be covered. Links to proof can be made, for example proving the area of a triangle. Students should be encouraged to write down any formulae they will be using before substituting in the numbers. Students should be able to solve questions in various contexts; these could include coordinate geometry or real-life situations. Questions may involve bearings, which may not be well remembered from GCSE so should be reviewed. Students should be encouraged to check that their answers are realistic as this check can show up errors. When completing multi-step questions emphasise to students that they should show all working out and use the answer function on their calculators to avoid rounding errors. It can be a useful teaching point to divide the class asking one side to round all answers and the other to keep values stored in their calculator to show how this affects the final answer. The unit circle can again be used to show how the trigonometric graphs are formed. Characteristics such as the period and amplitude should be discussed. Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos(x + 30)$, $y = \tan 2x$ is expected so this is a good opportunity to recap transformations.

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TRIGONOMETRIC IDENTIES AND EQUATIONS

Chapter 10 - Trigonometric Identities & Equations By the end of the sub-unit, students should: • be able to solve trigonometric equations within a given interval • understand and be able to use $\tan\theta = \frac{\cos\theta}{\sin\theta}$ • Understand and use sin $\sin^2\theta + \cos^2\theta = 1$	When solving trigonometric equations, finding multiple values within a range can initially be illustrated using the graphs of the functions. The decision can then be made whether to move on to using CAST diagrams or continue using graphs. Whichever method is used students will need plenty of practice in identifying all values within the limits correctly. Intervals with negative solutions as well as positive solutions should be used. Students should be able to solve equations such as $\sin(x + 70°) = 0.5$ for $0 < x < 360°$; $3 + 5\cos 2x = 1$ for $-180° < x < 180°$; and $6\cos^2 x + \sin x - 5 = 0$ for $0 < x < 360°$, giving their answers in degrees. Students should be comfortable factorising quadratic trigonometric equations and finding all possible solutions. It should be noted that in some cases only one of the factorisations will give solutions but in most case there will be two sets of solutions. Situations where one answer is equal to zero can cause some confusion with students then not looking for further solutions. This sort of example should be covered in class. For example, the equation, $\sin\theta(3\sin\theta + 1) = 0$ will often be simplified to just $3\sin\theta + 1 = 0$, resulting in the loss of solutions to the original equation.

VECTORS

Chapter 11 – Vectors (Part A Magnitude/Direction Addition & Scalar Multiplication) By the end of the sub-unit, students should: • be able to use vectors in two dimensions; • be able to calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form; • be able to add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.	Students need to be familiar with column vectors and with the use of **i** and **j** vectors in two dimentions. Students should be able to find a unit vector in the direction of ***a***, and be familiar with the notation $\|\boldsymbol{a}\|$. The triangle and parallelogram laws of addition should be known and students should be able to use them. Students should understand that vectors are commutative. Where answers are given in surds they should be simplified if possible. When performing operations on vectors this should also be understood geometrically, diagrams will be helpful here. Students should be able to use given diagrams but also draw their own in order to assist with questions. Students should understand and be able to use the conditions for parallel vectors. Use the classroom floor as a 2-dimensional grid to help students visualise vectors. Use the position of students in the room to illustrate concepts. Consider vectors in the real world, e.g. ask students to think of everyday phenomena that have a magnitude and direction e.g. forces, velocities, displacements.
Chapter 11 – Vectors (Part B Position Vector, Distance between two Points) By the end of the sub-unit, students should: • understand and be able to use position vectors; • be able to calculate the distance between two points represented by position vectors; • be able to use vectors to solve problems in pure mathematics and in context, (including forces).	Students should know and be able to use $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ Students should be able to calculate the distance between two points (x_1, y_1) and (x_2, y_2) using the formula $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$. Use the ratio theorem to find the position vector of a point *C* dividing *AB* in a given ratio. Use familiar shapes to illustrate the difference between 2 vectors and vector addition, e.g. parallelogram, rectangle. When solving problems using vectors only pure contexts are covered.

Chapter/Objectives	Teaching Points

DIFFERENTIATION	
<u>Chapter 12 – Differentiation (Part A First and Second Derivatives)</u> By the end of the sub-unit, students should: • understand and be able to use the derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a general point (x, y); • understand the gradient of the tangent as a limit and its interpretation as a rate of change; • be able to sketch the gradient function for a given curve; • be able to find second derivatives; • understand differentiation from first principles for small positive integer powers of x; • be able to differentiate x^2, for rational values of *n*, and related constant multiples, sums and differences.	Students should know that $\frac{dy}{dx}$ is the rate of change of y with respect to x. Knowledge of the chain rule is not required. The notation $\mathrm{f}'(x)$ may be used for the first derivative and $\mathrm{f}''(x)$ may be used for the second order derivative. Students should be able to identify maximum and minimum points as points where the gradient is zero. Cover the use of the second derivative to establish the nature of a turning point. Students should be able to sketch the gradient function $\mathrm{f}'(x)$ for a given curve $y = \mathrm{f}(x)$, using given axes and scale. This could involve speed and acceleration for example. Students should know how to differentiate from first principles. Students should be able to use, for $n = 2$ and $n = 3$, the gradient expression $$\lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$$ The alternative notations $h \to 0$ rather than $\delta x \to 0$ are acceptable. Students will need to be confident in algebraic manipulation of functions to ensure that they are in a suitable format for differentiation. For example, students will be expected to be able to differentiate expressions such as $(2x+5)(x-1)$ and $\frac{x^2+3x-5}{4x^{\frac{1}{2}}}$, for $x > 0$. Mistakes are easily made with negative and/or fractional indices so there should be plenty of practice with this.
<u>Chapter 12 – Differentiation (Part B Gradient, Tangents, Normals, Maxima & Minima)</u> By the end of the sub-unit, students should: • be able to apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points; • be able to identify where functions are increasing or decreasing.	Students should be able to use differentiation to find equations of tangents and normal, at specific points on a curve. This reviews and extends the earlier work on coordinate geometry. Maxima, minima and stationary points can be used in curve sketching. Problems may be set in the context of a practical problem. This could bring in area and volume from GCSE GCSE (9-1) Mathematics as well as using trigonometry. Students will need plenty of practice at setting up equations from a given context, in some cases this may include showing that it can be written in a particular form. Where students are given the answer to work towards they must be aware that they need to work forwards showing all steps clearly rather than starting with the answer and working backwards. Students need to know how to identify when functions are increasing or decreasing. For example, given that $\mathrm{f}'(x) = x^2 - 2 + \frac{1}{x^2}$, prove that $\mathrm{f}(x)$ is an increasing function. Use graph plotting software that allows the derivative to be plotted so that students can see the relationship between a function and its derivative graphically.

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INTEGRATION	
<u>Chapter 13 – Integration (Part A Indefinite Integrals)</u> By the end of the sub-unit, students should: • know and be able to use the Fundamental Theorem of Calculus; • be able to integrate x^n (excluding *n* = −1), and related sums, differences and constant multiples.	Integration can be introduced as the reverse process of differentiation. Students need to know that for indefinite integrals a constant of integration is required. Similarly to differentiation, students should be confident with algebraic manipulation. For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Introduce students to the integral sign; this can be useful in setting work out clearly on these sorts of questions and will be used later in definite integration. Given $\mathrm{f}'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = \mathrm{f}(x)$.
<u>Chapter 13 – Integration (Part B Definite Integrals & Area Under Graph)</u> By the end of the sub-unit, students should: • be able to evaluate definite integrals; • be able to use a definite integral to find the area under a curve.	It is important that students show their working out clearly as mistakes are easily made when putting values into a calculator. Students should also be encouraged to check their answers. Calculators that perform numerical integration can be used as a check, but a full method will be needed. Students will be expected to understand the implication of a negative answer from indefinite integration. Links can be made with curve sketching in questions where students need to find the points of intersection with the *x*-axis for a curve in order to find the limits of integration. Areas can be made up of a combination of a curve and a line so further links can be made to coordinate geometry.

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EXPONENTIALS AND LOGARITHMS	
Chapter 14 - Exponentials and logarithms By the end of the sub-unit, students should: • know and be able to use the function a^x and its graph, where a is positive; • know and be able to use the function e^x and its graph; • know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications; • know and be able to use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$; • know and be able to use the function $\ln x$ and its graph; • know and be able to use $\ln x$ as the inverse function of e^x; • understand and use the laws of logarithms: $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $k\log_a x - \log_a x^k$ (including, for example $k = -1$ and $k = -\frac{1}{2}$); • be able to solve equations of the form $a^x = b$; • be able to use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y;	When sketching the graph of a^x students should understand the difference in shape between $a < 1$ and $a > 1$. Explain to students that e^x is a special case of a^x. Graphs of the function e^x should include those in the form $y = e^{ax+b} + c$. Students should realise that when the rate of change is proportional to the y-value, an exponential model should be used. An ability to solve equations of the form $e^{ax+b} = p$ and $\ln\ (ax + b) = q$ is expected. Students can use the laws of indices to prove the laws of logarithms and show that $log_a a = 1$. In solving equations students may use the change of base formula. Solving equations questions may be in the form $2^{3x-1} = 3$. Students should be able to plot log y against log x and obtain a straight line where the intercept is $\log_a$ and the gradient is n and plot log y against x to obtain a straight line where the intercept is log k and the gradient is log b. There should be discussion about why this is an appropriate model and why it is only an estimate. Contexts for modelling should could include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth. Students should be familiar with terms such as initial, meaning when $t = 0$. They may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required.
• understand and be able to use exponential growth and decay in modelling, giving consideration to limitations and refinements of exponential models	