Department of Mathematics
Year 12 Scheme of Work – Mechanics

3 – 2 = 1 → $\sin^2\theta + \cos^2\theta = 1$ → $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2}\,dt = 1$

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MODELLING IN MECHANICS

Chapter 8 – Modelling in Mechanics (Part A Standard S.I. Units of Length, Time and Mass) By the end of the sub-unit, students should: • understand the concept of a mathematical model, and be able to abstract from a real-world situation to a mathematical description (model); • know the language used to describe simplifying assumptions; • understand the particle model; • be familiar with the basic terminology for mechanics; • be familiar with commonly-made assumptions when using these models; • be able to analyse the model appropriately, and interpret and communicate the implications of the analysis in terms of the situation being modelled; • understand and use fundamental quantities and units in the S.I. system: length, time and mass; • Understand that units behave in the same way as algebraic quantities, e.g. meters per second is $\mathrm{m/s} = \mathrm{m} \times 1/\mathrm{s} = \mathrm{ms}^{-1}$	Begin by asking students 'What is mechanics?' Lead them to the idea that mechanics is a branch of applied mathematics that deals with motion and the forces producing motion. Students need to be comfortable with the idea that mathematics is used to model real life and need to become familiar with the modelling cycle: mechanics problem → create a mathematical model (using diagrams, general principles or formulae) → solve the model → refer back to the original problem → refine the model [Link with the data-handling cycle] It is important for students to get a 'feel' for mechanics at this early stage in order to support later work.
Chapter 8 – Modelling in Mechanics (Part B Definitions) By the end of the sub-unit, students should: • understand and use derived quantities and units: velocity, acceleration, force, weight; • know the difference between position, displacement and distance; • know the difference between velocity and speed, and between acceleration and magnitude of acceleration; • know the difference between mass and weight (including gravity); • understand that there are different types of forces.	Revise GCSE (9-1) in Mathematics compound units for speed and acceleration and make sure that students are comfortable converting from one unit to another, e.g. from km h^{-1} into ms^{-1}. Define the vector quantities displacement and velocity as the vector versions of distance and speed respectively. Begin by walking across the room and explaining the difference between position (referred to a fixed origin), displacement (a vector measured from any position) and distance (a scalar quantity for the total movement). Then move onto discussing speed (the rate at which an object covers distance) and velocity (the rate of change of displacement or speed in a certain direction. Mention the special acceleration (for a falling object) due to gravity. In this course, this value is assumed to be a constant *g*, usually 9.8 m s^{-2} though it does vary in the real world. This could be a good opportunity to dispel common misconceptions around weight and mass. Make it clear that mass is the amount of 'stuff' something is made of, is a scalar and is fixed (in kg), whereas weight is a force of attraction between an object and the centre of the earth and can vary depending on gravity and is measured in newtons. Hence weight = mass × gravity (or $W = mg$).

Chapter/Objectives	Teaching Points

CONSTANT ACCELERATION

Chapter 9 – Constant Acceleration (Part A Graphical Representation) By the end of the sub-unit, students should: • be able to draw and interpret kinematics graphs, knowing the significance (where appropriate) of their gradients and the areas underneath them.	Introduce this topic by making links to the GCSE (9-1) in Mathematics prior knowledge for distance-time (travel) and speed-time graphs. Kinematics is the analysis of a particle's motion without reference to the resultant force that caused that motion. Stress that forces causing the motion of the body in this section are *constant*, therefore acceleration is constant and this results in a *straight line* travel speed-time or velocity-time graph. Extend the ideas to displacement by considering a particle which moves in reverse direction back beyond the starting point. For a velocity-time graph, consider the units for the area of a unit square 1 m s^{-1} by 1 s. The 's' cancels, leaving 'm', therefore the area represents the displacement. Discuss and interpret graphs that model real situations. For example, the distance-time graph for a particle moving with constant speed, the velocity-time graph for a particle with constant acceleration.
Chapter 9 – Constant Acceleration (Part B SUVAT Formulae) By the end of the sub-unit, students should: • recognise when it is appropriate to use the *suvat* formulae for constant acceleration; • be able to solve kinematics problems using constant acceleration formulae; • be able to solve problems involving vertical motion under gravity.	Make links back to Unit 7a and contrast the previous graphical approach with this algebraic approach. Note that there are five quantities, *s, u, v, a* and *t* (four vectors and one scalar) and each formula relates four of them hence there are five formulae. The formulae that must be derived and learnt are: • $v = u + at$ • $s = \frac{(u+v)t}{2}$ • $s = ut + \frac{1}{2}at^2$ • $v^2 = u^2 + 2as$ • $s = vt - \frac{1}{2}at^2$ These formulae are only valid for *constant* acceleration in a straight line (and are referred to as the *suvat* formulae). When solving problems, write down known variables and the variable(s) to be found - this should help to identify which one (or more, as some problems will involve simultaneous equations) of the *suvat* formulae to select. Emphasise to students the need to make sure units are compatible.
	Model the good practice of drawing a diagram to illustrate the situation whenever possible, especially when considering vertical motion under gravity. This will encourage students to draw their own diagrams. Mark the positive direction on the diagram and take acceleration due to gravity (*g*) to be 9.8 m s^{-2} unless directed otherwise. Students may assume that *g* is constant, but they should be aware that *g* is not a universal constant but depends on location. If an object is thrown upwards and upwards is taken as being positive then a = -9.8 m s^{-2}. Explain that the velocity is zero at the greatest height and there is symmetry in the path (up and down to the same point) due to the fact that we model air resistance as being negligible.

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FORCES AND MOTIONS

Chapter 10 – Forces and Motion (Part A Newton's First Law) By the end of the sub-unit, students should: understand the concept of a force; understand and use Newton's first law.	Relate this topic back to the different types of forces defined in Unit 6b. Newton said '*An object continues in state of rest or uniform motion unless acted on by an external force*.' Hence one can define a force as something which causes a body to accelerate. Explain to students that 'no force acting' means a body will either be stationary or be moving with constant velocity (i.e. acceleration = zero). This is why in outer space an object keeps moving at constant speed once pushed (there are no forces to speed it up, slow it down or stop it moving.) So, an object at rest or constant velocity ⇒ no resultant force; an object changing speed or direction ⇒ resultant force. This will lead to Newton's second law in the next section. Newton also stated '*When an object A exerts a force on another object B there is an equal and opposite reaction force of B on A.*' Explain that if a book is on a smooth, horizontal table, the forces acting on the book are the Weight, *W* (vertically down) and the normal reaction, *R* (always at 90° to the *surface* of contact). Assuming the table surface material is strong enough to hold the full weight of the book, the two forces balance each other and there is no resultant force. The book does not move, hence it is in equilibrium. Ask questions such as: If the book has a mass of 5 kg, what is its weight? Therefore, what would the magnitude of the normal reaction be to guarantee equilibrium? Draw different examples of force diagrams to illustrate: weight, reaction, tension (in strings), thrust (in rods), compression (in light rods, springs) etc. To illustrate thrust, balance a book on a ruler.In which direction is the thrust force acting? Introduce the **i - j** notation. The forces can be given in **i - j** form or as column vectors. Questions on equilibrium will be limited to perpendicular forces so the sum of the forces must be 0**i** + 0**j** for equilibrium.
Chapter 10 – Forces and Motion (Part B Newton's Second & Third Laws) By the end of the sub-unit, students should: • understand and be able to use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D (**i**, **j**) vectors.);	Newton stated, '*Where there is a force, there is an acceleration (or deviation from uniform motion) and the force is proportional to the acceleration*'. Therefore $F \propto a$, and choosing the constant to suit the motion units gives $F = ma$. (Newton's second law). This is known as the 'equation of motion'. Explain to students that if they sum all the effects of the forces acting, in a particular direction, this will be equal to the mass x the acceleration in that direction. This process is called resolving the forces in that direction e.g. resolving horizontally, or $R(\rightarrow)$ for short. It's usually best to resolve IN the direction of the acceleration and/or perpendicular to the direction of the acceleration. When resolving always take the positive direction as the direction of the acceleration and put all the forces on one side of the equation and (mass x acceleration) on the other side.
• understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles.	When working on connected particles problems (such as trains or pulley systems) explain to students that they should consider the whole system as well as the separate parts. Applications to be covered are lift problems, car and caravan type questions and connected particles passing over a smooth pulley. Consider both pulley scenarios: a pulley with both stings hanging vertically; and a pulley at the end of a horizontal table.

Chapter/Objectives	Teaching Points
VARIABLE ACCELERATION	
Chapter 11 – Variable Acceleration (Part A Calculus & Rates if Change) By the end of the sub-unit, students should: • be able to use calculus (differentiation) in kinematics to model motion in a straight line for a particle moving with variable acceleration; • understand that gradients of the relevant graphs link to rates of change; • know how to find max and min velocities by considering zero gradients and understand how this links with the actual motion (i.e. acceleration = 0).	Start by stating that the *suvat* formulae from Unit 7 can only be used when acceleration is constant and the motion is in a straight line. This means the speed-time or velocity-time graphs are made up of straight lines. Draw the graph of say, $v = 2t^2 + 2t + 1$ (for $t > 0$). This is part of a parabola where the gradient is increasing so as time passes the object is accelerating more quickly. As acceleration is not constant, the *suvat* formulae will not work for this model. Make links (using AS Pure Mathematics calculus) to the rate of change of velocity explaining that $\frac{dv}{dt}$ = gradient = acceleration. This idea that the gradient of a velocity-time graph gives acceleration should be familiar from previous work in Unit 7 and also from GCSE (9-1) in Mathematics. Summarise the situation by talking about, velocity as the rate of change of displacement and acceleration as the rate of change of velocity. Express these statements in the notation of calculus: $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. Students will also need to relate the fact that the gradient = 0 at the max or min point to this mathematical model i.e. if $\frac{dv}{dt}$ = 0, then acceleration = 0, so the particle must be at max or min velocity, as it cannot accelerate (or get any faster or slower) any more at this point in time.
Chapter 11 – Variable Acceleration (Part B Integration & Kinematics Problems) • By the end of the sub-unit, students should: • be able to use calculus (integration) in kinematics to model motion in a straight line for a particle moving under the action of a variable force; • understand that the area under a graph is the integral, which leads to a physical quantity; • know how to use initial conditions to calculate the constant of integration and refer back to the problem.	Return to the graph of $v = 2t^2 + 2t + 1$ (for $t > 0$) introduced at the start of Unit 9a. From earlier work in Unit 7 and from GCSE (9-1) in Mathematics, students should know that the area under a velocity-time graph equals the displacement. Remind students that, from their work for Pure Mathematics, the area under a curve can be found using integration. This means that the integral of the velocity expression (with respect to time) gives the displacement. By linking integration with the reverse of differentiation, displacement and velocity can be found by integrating expressions for velocity and acceleration respectively: $r = \int v\mathrm{d}t$ and $r = \int a\mathrm{d}t$ (Again '*s*' can be used in place of '***r***' for straight line motion in this section) Move on to explain that the constant of integration, c needs to be found by referring back to the problem and using some (usually initial) information about the body. For example knowing that the particle starts from *O* at rest means that when $t = 0$ (initially), $s = 0$ (at *O*) and $v = 0$ (at rest). These values can be substituted to calculate c.