Boolean Algebra

Induction

Summer 2020

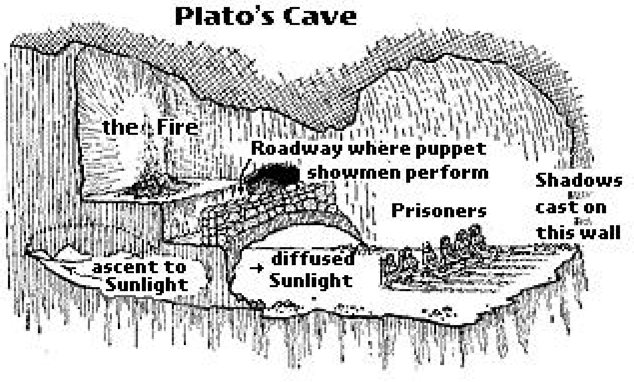
Boolean Algebra

- Main Focus:
  - Boolean Algebra operations

Bit of History

- George Boole come up with the first approach to modern logic, the original studies done by Plato followed by Aristoteles.

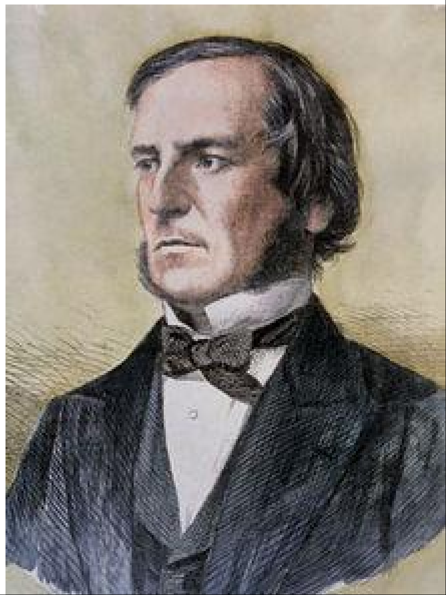
- It is widely accepted that Plato's Cave is the first study into logic



Boole

- George Boole decided to approach the ideas and bring them to a modern society. Being a philosopher this helped greatly and George Boole notation and work on the Logic field is now seen as a vital point in mathematics history.

- And he has no idea how useful it become!



Atomic Sentences

- Although we do not need to cover these it is important to note that in Computing we talk about A, B and C being inputs but if you do some uni material research you will come upon the concept of Atomic Sentences

Atomic Sentences

- Atomic sentences are constructs which can only be true or false – they accept no grey areas
- For example
- “We are at this moment in time all in Room 16”

- Please do not use Physics on this matter, Philosophy will always stay above mundane concepts!

Inputs?

- In Computing A,B and C will be seen as inputs.

- The correlation here is that like atomic sentences they accept no grey areas and therefore only accept true or false or for us 1 or 0

Boolean Constructs

- Boole, having Plato’s and Aristoteles work in mind derived Logic terms which we need to be familiar with:
  - AND
  - OR
  - NOT
  - NAND
  - NOR
  - EXOR

I will cover all of them with both European and North American symbols in case you come across different sources!

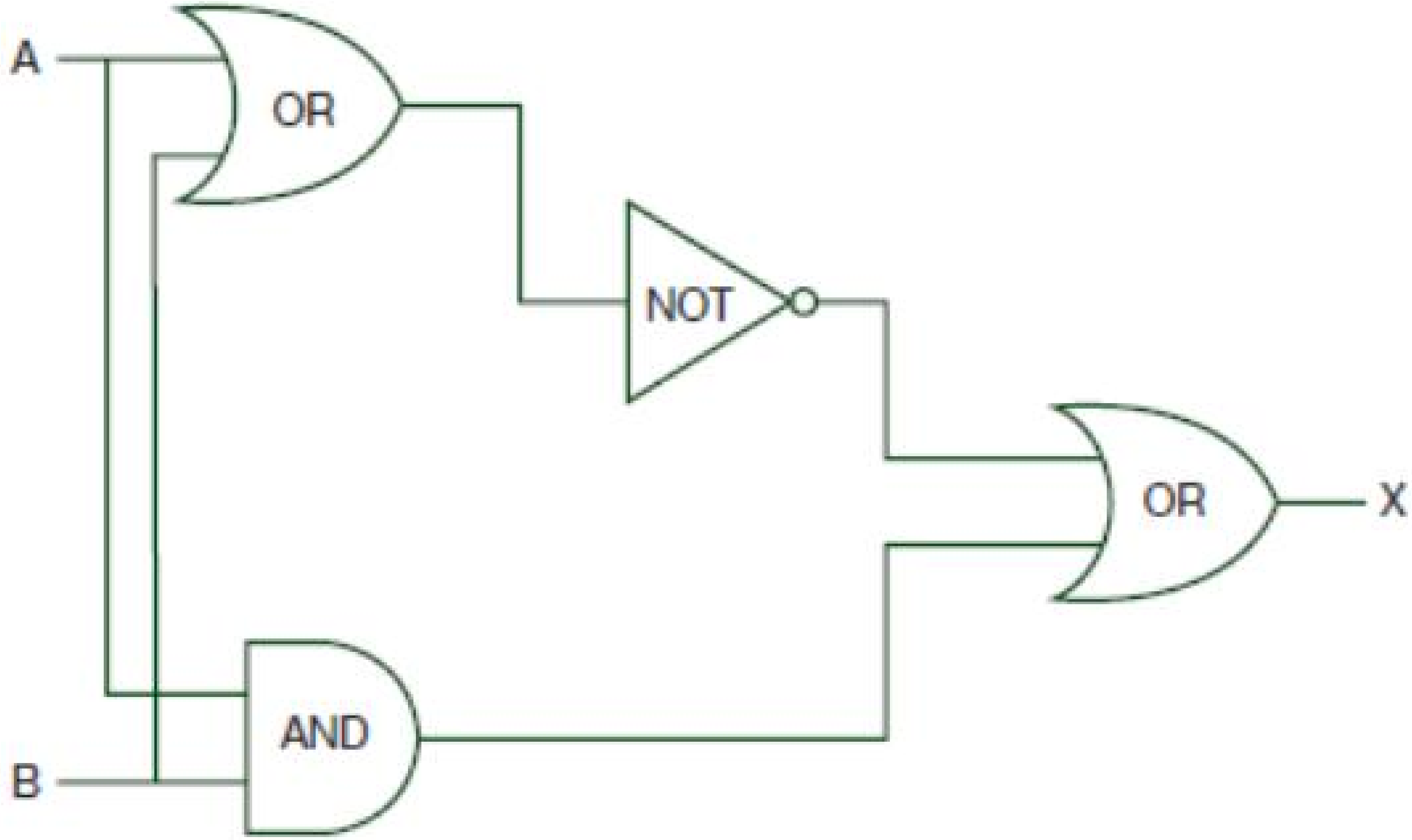
Special cases

- Tautology:
  - When all inputs are always true (1)

- Fallacy:
  - When all inputs are always false (0)

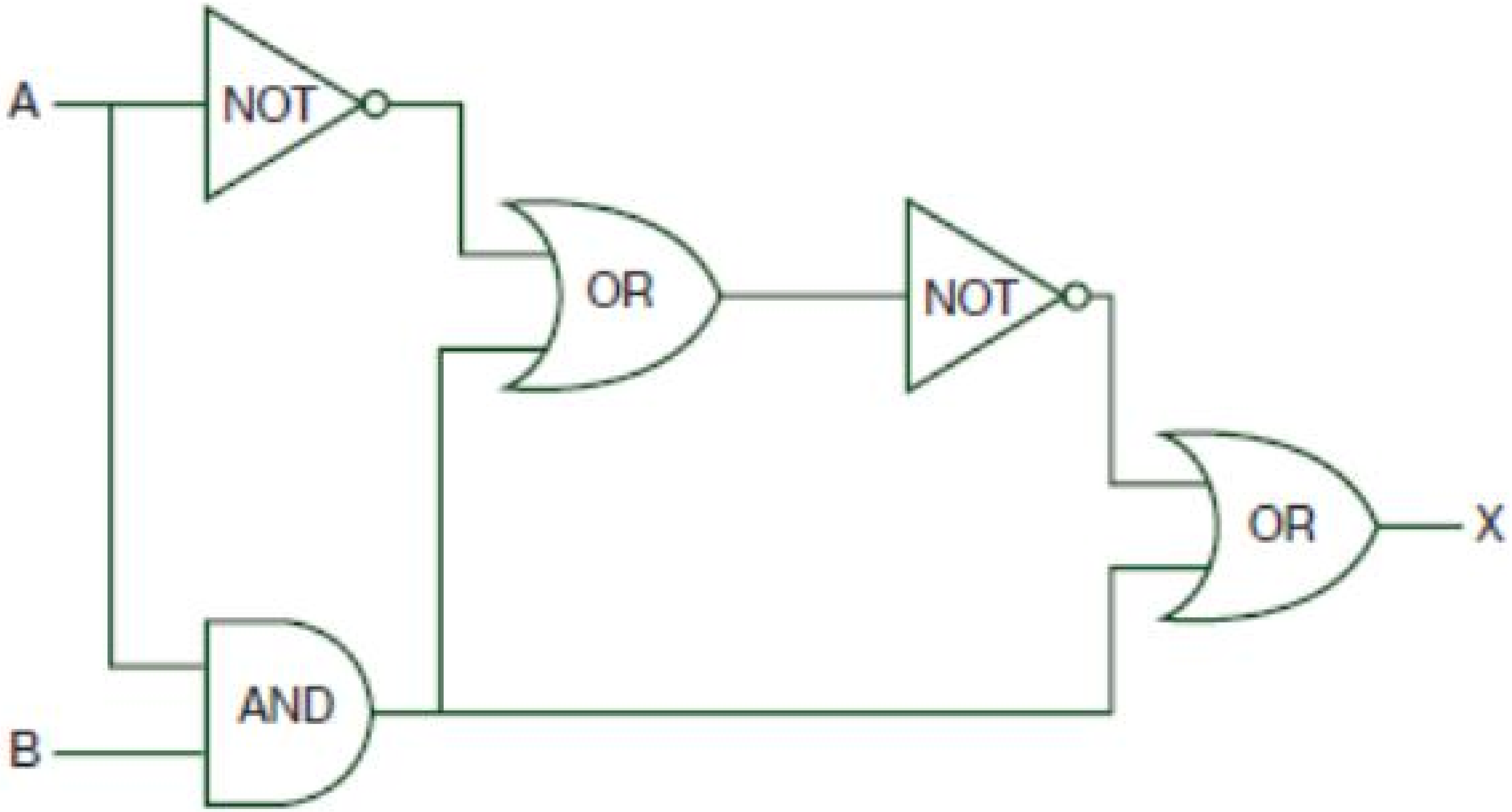
Exercises

9.4.1



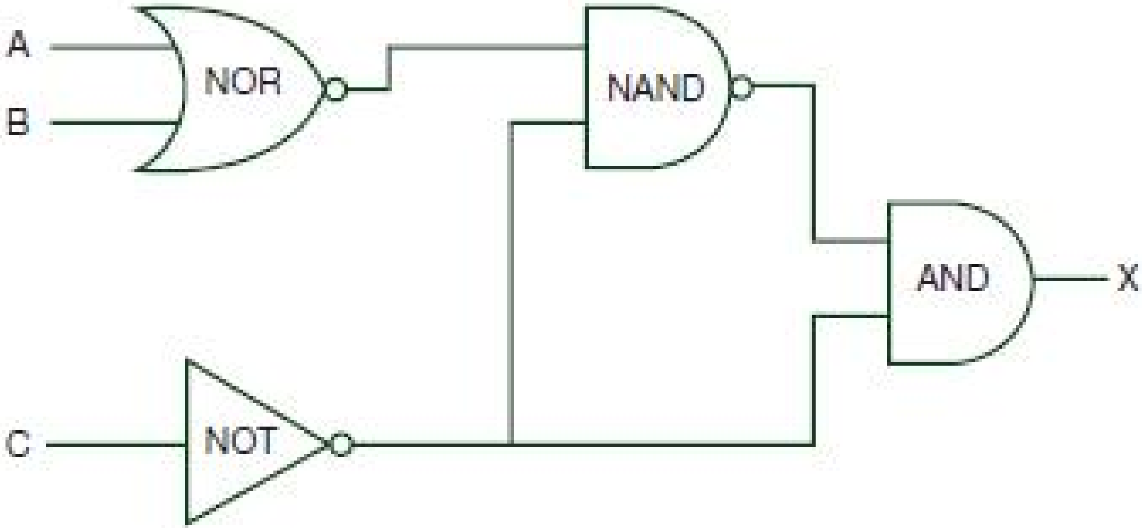
Exercises

9.4.2

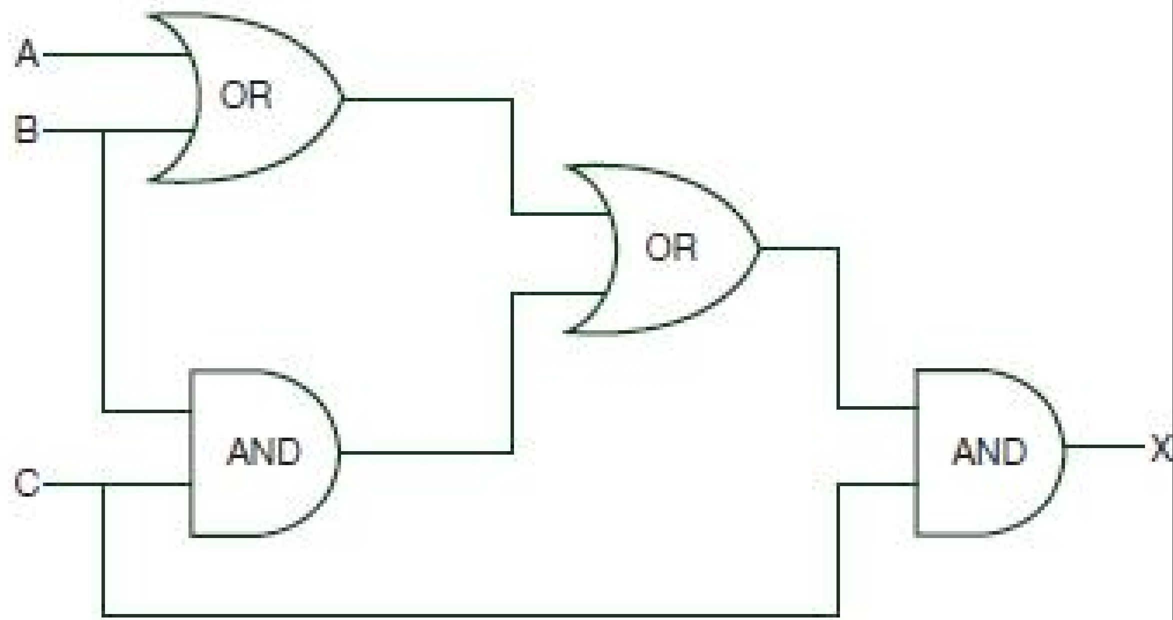


Exercises

9.4.3

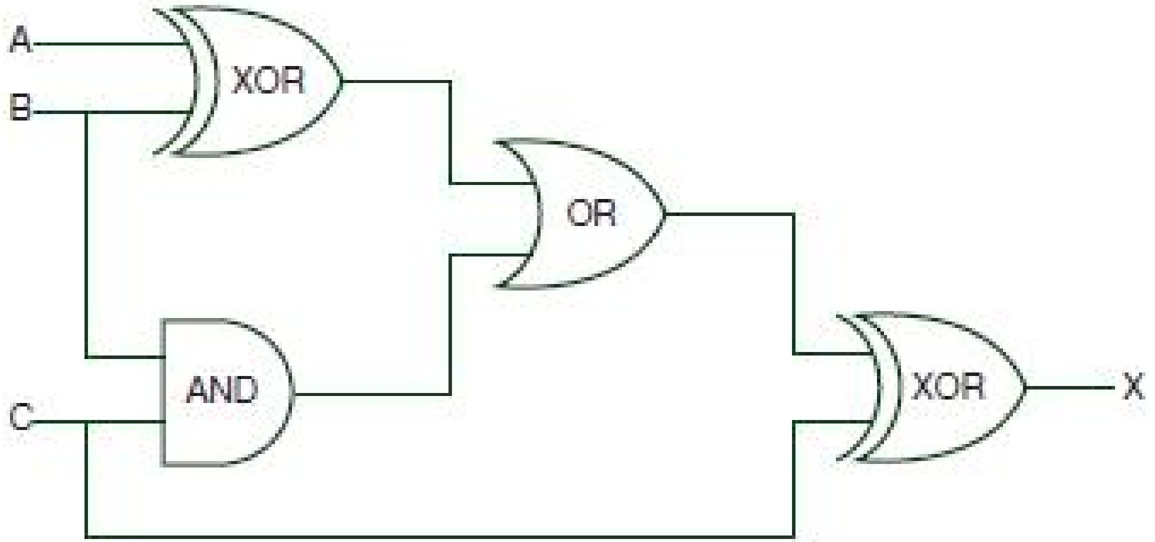


9.4.4

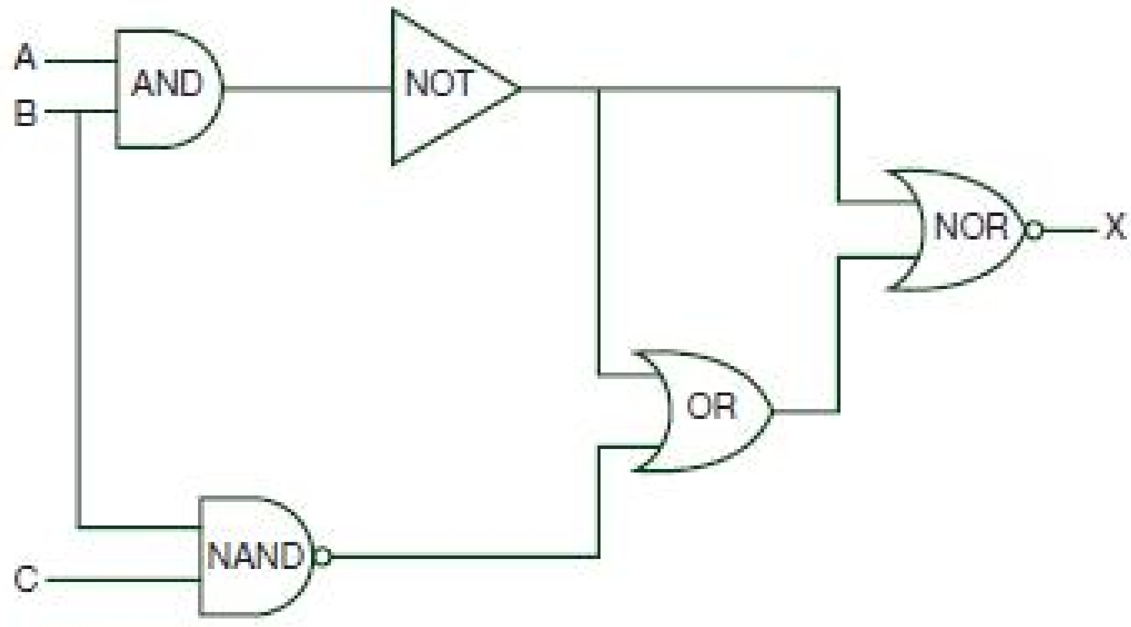


Exercises

9.4.5

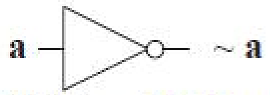


9.4.6



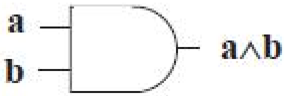
Revision!!!

Input a	Output ~a
0	1
1	0



This is a **NOT** *gate*

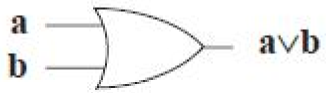
Input a	b	Output a∧b
0	0	0
0	1	0
1	0	0
1	1	1



This is an **AND** *gate*

Revision

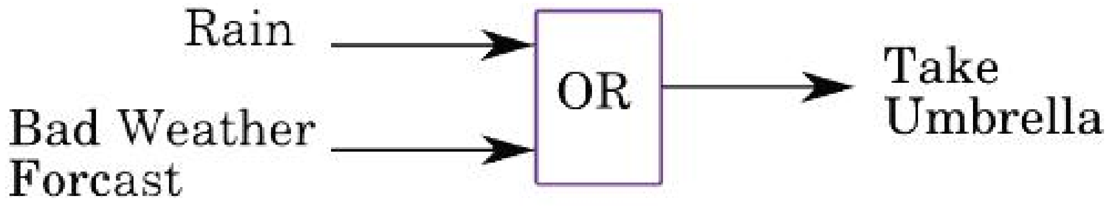
Input		Output
a	b	a∨b
0	0	0
0	1	1
1	0	1
1	1	1



This is an **OR** *gate*

A simple example

"I will take an umbrella with me if it is raining or the weather forecast is bad"

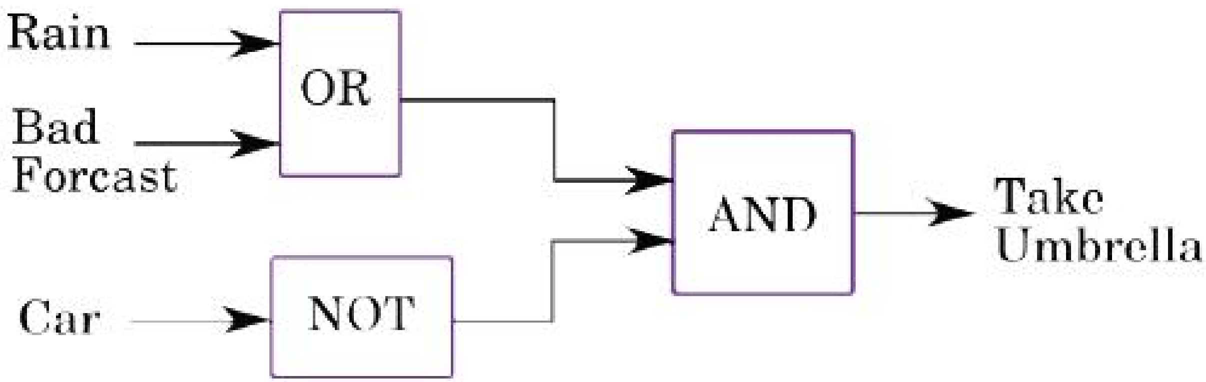


Raining	Bad Forecast	Umbrella
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	TRUE

Making it more complex!!!

"If I do not take the car then I will take the umbrella if it is raining or the weather forecast is bad"

$$(\textit{Take Umbrella}) = (\text{ NOT } (\textit{Take Car})) \text{ AND } ((\textit{Bad Forecast}) \text{ OR } (\textit{Raining}))$$



In a more Mathematical way

$$U = C' \cdot (W + R)$$

R	W	C	$X_1 = R + W$	$X_2 = C'$	$U = X_1 \cdot X_2$
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

Boolean Algebras

- Using the Unary Operator **NOT**

$$(A')' = A$$
$$A \cdot A' = 0$$
$$A + A' = 1$$

Boolean Algebras

- Distributive, Associative and Commutative

Associative	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
	$(A + B) + C = A + (B + C)$
Commutative	$A \cdot B = B \cdot A$
	$A + B = B + A$
Distributive	$A \cdot (B + C) = A \cdot B + A \cdot C$
	$A + (B \cdot C) = (A + B) \cdot (A + C)$

Boolean Algebras

- Simplification and Using Constants

$$A \cdot A = A$$
$$A + A = A$$

$$A \cdot 0 = 0$$
$$A \cdot 1 = A$$
$$A + 0 = A$$
$$A + 1 = 1$$

Boolean Algebras

- Exercise:
- Prove that $A + A \cdot (B) = A$

De Morgan Rules

- Special rules in Boolean Algebra

$$(A+B)' = A' \cdot B'$$
$$(A \cdot B)' = A' + B'$$

- These are valid for n arguments

$$(A+B+C)' = ((A+B)+C)' = ((A+B)') \cdot C' = A' \cdot B' \cdot C'$$

$$(A \cdot B \cdot C \cdot .... \cdot X)' = A' + B' + C' + ... + X'$$